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# The minimal $\mathcal{N}=4$ no-scale model from generalized dimensional reduction

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Abstract: We consider the generalized dimensional reduction of pure ungauged $\mathcal{N}=4$, $D=5$ supergravity, where supersymmetry is spontaneously broken to $\mathcal{N}=2$ or $\mathcal{N}=0$ with identically vanishing scalar potential. We explicitly construct the resulting gauged $D=4$ theory coupled to a single vector multiplet, which provides the minimal $\mathcal{N}=4$ realization of a no-scale model. We discuss its relation with the standard classification of $\mathcal{N}=4$ gaugings, extensions to non-compact twists and to higher dimensions, the $\mathcal{N}=2$ theories obtained via consistent $\mathcal{Z}_2$ orbifold projections and prospects for further generalizations.

Keywords: Field Theories in Higher Dimensions, Supersymmetry Breaking, Extended Supersymmetry, Supergravity Models.



 

## Contents



## 1. Introduction

Generalized dimensional reductions of higher-dimensional supergravity [1] and superstring theories [2] provide elegant and efficient mechanisms for supersymmetry breaking. In short, the periodicity conditions in the compact extra dimensions can be 'twisted' by an $R$-symmetry of the higher-dimensional action. This induces four-dimensional gravitino mass terms and breaks the supersymmetries that do not commute with the twist, without generating a four-dimensional vacuum energy. Also M-theory and superstring compactifications with branes, orientifolds and fluxes (for a review and an extensive list of references, see e.g. [3]) can be related with twisted tori by suitable dualities [4, 5]. The above theoretical constructions can play a rôle in solving some crucial problems of higher-dimensional supergravity and superstring theories: vacuum selection, supersymmetry breaking, moduli stabilization, generation of hierarchies. The corresponding $D=4$ effective theories are



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gauged extended ($\mathcal{N} > 1$) supergravities,[1] where a subgroup of the isometry group of the scalar manifold (which includes the $R$-symmetry group) is promoted to a local invariance.

It has been known for quite some time that generalized dimensional reductions of $D = 11$ [6] or minimal $D = 10$ [7] supergravity give rise to gauged $\mathcal{N} = 8$ [8] or $\mathcal{N} = 4$ [9] $D = 4$ supergravities, and that daughter theories with a lower number of supersymmetries can be obtained by suitable orbifold projections. Systematic investigations of the effective gauged $D = 4$ theories have recently been performed starting from $\mathcal{N} = 8$ [10, 11] or $\mathcal{N} = 2$ [12] supergravities; many results have been obtained also for the $\mathcal{N} = 4$ theories [13, 14] that are the focus of the present paper. Indeed, even in the case of $\mathcal{N} = 1$ orbifolds of superstrings or M-theory, the effective $D = 4$ superpotential for the light (bulk) states coming from the untwisted sector can be associated with the gauging of the underlying $\mathcal{N} = 4$ theory [15].

The goal of the present paper is to consider generalized dimensional reductions of pure $\mathcal{N} = 4$, $D = 5$ ungauged supergravity [16, 17], and to derive explicitly lagrangian and transformation laws for the resulting low-energy theory, a gauged $\mathcal{N} = 4$, $D = 4$ supergravity coupled to a single vector multiplet. Such a theory can be legitimately called the minimal $\mathcal{N} = 4$ no-scale model, in analogy with the minimal no-scale models with $\mathcal{N} = 1$ [18] and $\mathcal{N} = 2$ [19] constructed long ago. It is also the minimal $\mathcal{N} = 4$ no-scale model that allows for the partial breaking to $\mathcal{N} = 2$, whereas partial breaking to $\mathcal{N} = 3$ or $\mathcal{N} = 1$ requires the presence of additional vector multiplets [20]: we will see that the minimal $\mathcal{N} = 2$ no-scale model with partial breaking [21], which contains one vector multiplet and one hypermultiplet, corresponds to a consistent $\mathcal{Z}_2$ orbifold projection of our minimal $\mathcal{N} = 4$ no-scale model [22]. The choice of $\mathcal{N} = 4$ for our detailed study is motivated not only by the need of filling a gap in the existing literature, but also by the fact that $\mathcal{N} = 4$ is the minimal amount of supersymmetry shared by all stable superstring theories, and their corresponding supergravities, in $D = 10$. Within $\mathcal{N} = 4$ theories, the choice of $D = 5$ as the starting point for the generalized dimensional reduction allows for minimality (when no $D = 5$ vector multiplets are included) as well as for maximum generality (when an arbitrary number of $D = 5$ vector or tensor multiplets are included), and prepares the ground for further discussions of the minimal Randall-Sundrum model [23] with extended supersymmetry [24], which is based on a pure gauged $\mathcal{N} = 4$, $D = 5$ supergravity [17, 25].

The paper is organized as follows. In section 2 we briefly recall the field content and the symmetries of pure, ungauged $\mathcal{N} = 4$, $D = 5$ supergravity. In section 3 we derive the ungauged $\mathcal{N} = 4$, $D = 4$ effective supergravity, coupled to a single vector multiplet, that originates from standard dimensional reduction. In particular, we identify the 'electric' subgroup of the $D = 4$ duality group that acts linearly on the gauge potentials and on the scalar fields. The gauged $\mathcal{N} = 4$, $D = 4$ supergravity, originating from the pure ungauged $D = 5$ theory by generalized dimensional reduction, is derived in section 4. Twisting the periodicity conditions by an $R$-symmetry transformation, controlled by two independent parameters, does not generate a potential, and leads to $\mathcal{N} = 4$ no-scale models

[1]Throughout this paper, we shall count the number of supersymmetries in any dimension in terms of the equivalent number $\mathcal{N}$ of four-dimensional supersymmetries.



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with spontaneous breaking of half or all of the supersymmetries. As already stressed in similar contexts [20, 4, 10, 11], the effective $\mathcal{N} = 4$, $D = 4$ gauged supergravity does not fit the standard classification [26]: the reason is that formulations of the theory that are equivalent, in the ungauged case, via duality transformations, are no longer equivalent in the gauged case. For completeness, we also discuss the possibility of a non-compact twist, which generates a positive-definite four-dimensional potential without critical points [27]. In section 5, we explain how our results can be applied to generalized dimensional reductions of pure $\mathcal{N} = 4$ supergravity from $D$ to $D - 1$ dimensions, and we discuss the examples of the non-chiral (2,2) and the chiral (4,0) theories reduced from six to five dimensions. In the former case, we obtain new 'flat' gaugings of $\mathcal{N} = 4$ in $D = 5$, not included in the classification of [28], which was limited to gaugings of semisimple and abelian groups. In the latter case, we generalize the flat gauging of ref. [25] to partial breaking. In section 6 we discuss consistent $\mathcal{Z}_2$ orbifold truncations in the presence of the Scherk-Schwarz twist, and the main features of their $\mathcal{N} = 2$, $D = 4$ effective theories: we find total or partial supersymmetry breaking with vanishing potential, in agreement with previous results [21, 22, 12]. In the concluding section, we summarize again our results and we comment on prospects for further work. Our conventions are explicitly spelled out in appendix A. The lagrangian and the transformation laws of pure ungauged $\mathcal{N} = 4$, $D = 5$ supergravity are recalled in appendix B. The details of the $\mathcal{N} = 4$, $D = 4$ lagrangian and transformation laws, relevant to both standard and generalized dimensional reduction, are collected in appendix C.

## 2. $\mathcal{N} = 4$, $D = 5$ ungauged supergravity

The field content of pure $\mathcal{N} = 4$, $D = 5$ supergravity, whose ungauged version was first constructed in [16, 17], is just the gravitational multiplet: one graviton $g_{MN}$, four gravitinos $\psi_{M\,a}$, five plus one vectors $V_M^i + v_M$, four spin-1/2 fermions $\chi_a$ and one real scalar $\phi$. The automorphism group of the $\mathcal{N} = 4$, $D = 5$ supersymmetry algebra ($R$-symmetry) is USp(4). Fields with indices $a = 1, \ldots, 4$ and $i = 1, \ldots, 5$ transform in the **4** and **5** irreducible representations, respectively; all the other fields are singlets. In a schematic notation that will be convenient in the following, we summarize the content of the gravitational multiplet as:

$$ {}_{D=5}[1, 4, 5 + 1, 4, 1]^{\mathcal{N}=4}_{m=0} \,. \tag{2.1} $$

The numbers in brackets count the representations of different 'spin', with the latter decreasing from left to right in steps of 1/2. The subscript '${}_{m=0}$' recalls that we are dealing with a massless multiplet. Further details on our conventions can be found in appendix A. The lagrangian and the supersymmetry transformations are collected in appendix B.

Before moving to the study of standard and generalized dimensional reductions to $D = 4$, it is useful to recall the local and global symmetries of the $D = 5$ theory. The local symmetries are general coordinate transformations, $\mathcal{N} = 4$ supersymmetry and a $\mathrm{U}(1)^6$ gauge invariance, associated with the six vector fields and with respect to which no fields are charged. The global symmetry of the theory, or '$U$-duality' group, is in this case $\mathrm{USp}(4) \times \mathrm{SO}(1,1)$. It is an electric subgroup of the $D = 4$ $\mathrm{Sp}(14, \mathbb{R})$ duality group acting on the space of the vector field strengths and their duals [29], in the sense that it acts linearly on the vector potentials. The action of $\mathrm{SO}(1,1)$ on the fields is:

$$V_M^i \to e^{-\lambda}\, V_M^i\,, \qquad v_M \to e^{2\lambda}\, v_M\,, \qquad \phi \to \phi + \sqrt{6}\lambda\,, \tag{2.2}$$

while $\mathrm{USp}(4) \sim \mathrm{SO}(5)$ acts canonically on the $a$ and $i$ indices.

If we couple $\mathcal{N}=4$, $D=5$ pure supergravity to $n$ vector multiplets, the $U$-duality group is enlarged to $\mathrm{SO}(5,n)\times\mathrm{SO}(1,1)$, and the scalar manifold becomes:

$$\frac{\mathrm{SO}(5,n)}{\mathrm{SO}(5)\times\mathrm{SO}(n)} \times \mathrm{SO}(1,1)\,. \tag{2.3}$$

In this case, the combination of generators used for the twist and giving rise to a flat $D=4$ gauging can be embedded in the maximal compact subgroup of $\mathrm{SO}(5)\times\mathrm{SO}(n)$. However, switching on the $\mathrm{SO}(n)$ sector does not increase the possibilities for supersymmetry breaking via generalized dimensional reduction, it just increases the dimension of the $D=4$ gauge group.[2] Moreover, some of the resulting non-minimal $\mathcal{N}=4$ no-scale models have already been constructed as truncations of gauged $\mathcal{N}=8$ theories [13, 14]. For these reasons, in the following we will focus on the minimal model ($n=0$), corresponding to pure $\mathcal{N}=4$, $D=5$ supergravity.

## 3. Standard dimensional reduction

We now describe the salient features of the standard dimensional reduction of the $\mathcal{N}=4$, $D=5$ theory. For this purpose, we consider only the kinetic and Chern-Simons terms of the $D=5$ lagrangian eq. (B.1):

$$\begin{aligned} e_5^{-1}\mathcal{L}_5^{kin} &= -R_5 - \frac{1}{2}\partial_M\phi\,\partial^M\phi - \frac{1}{4}X^4 v_{MN}v^{MN} - \frac{1}{4}X^{-2}V^i_{MN}V_i^{MN} - \\ &-\frac{1}{8}e_5^{-1}\varepsilon^{MNRST}V^i_{MN}V^i_{RS}v_T - \frac{i}{2}\overline{\psi}^a_M\gamma^{MNR}D_N\psi_{Ra} + \frac{i}{2}\overline{\chi}^a\gamma^M D_M\chi_a\,. \end{aligned} \tag{3.1}$$

The first step is to decompose the fields with five-dimensional indices into fields with four-dimensional indices:

$$E_M^{\ A} = \begin{pmatrix} \rho^{-1/2}e_\mu^\alpha & \rho A_\mu \\ 0 & \rho \end{pmatrix}, \qquad \psi_{Ma} = \begin{pmatrix} \psi_{\mu a} \\ \psi_{y a} \end{pmatrix}, \qquad V_M^i = \begin{pmatrix} V_\mu^i \\ V_5^i \end{pmatrix}, \qquad v_M = \begin{pmatrix} v_\mu \\ v_5 \end{pmatrix}. \tag{3.2}$$

We can then assume that the zero modes do not depend on the fifth coordinate $y$ and perform the following field redefinitions:

$$\begin{aligned} \psi_\mu^a &= \rho^{-1/4}\eta_\mu^a + \left(A_\mu + \frac{i}{2}\rho^{-3/2}\widehat{\gamma}\gamma_\mu\right)\psi_y^a\,, \\ \psi_{y\,a} &= \rho^{5/4}\psi'_{y\,a}\,, \qquad \chi_a = \rho^{1/4}\chi'_a\,, \\ V_\mu^i &= B_\mu^i + V_5^i A_\mu\,, \qquad v_\mu = b_\mu + v_5 A_\mu\,, \\ t &= \rho\, X^{-2}\,, \qquad \tau = v_5\,, \qquad \varphi_0 = \sqrt{2}\,\rho\, X\,, \qquad \varphi^i = V_5^i\,. \end{aligned} \tag{3.3}$$

[2] More possibilities arise only when considering $\mathcal{N}=4$, $D=4$ gaugings [20, 10, 11, 13, 14] that cannot be originated just by Scherk-Schwarz reductions of $\mathcal{N}=4$ theories, but require instead generalized dimensional reductions of $\mathcal{N}=8$ theories combined with a $\mathcal{Z}_2$ orbifold that explicitly breaks half of the supersymmetries.



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The first two lines allow us to 'ortho-normalize' the $D = 4$ fermionic kinetic terms, the third one avoids mixing terms of the form '$V^{\mu\nu} A_\mu \partial_\nu \varphi$', and the last one makes the duality invariance of the scalar sector manifest.

After moving from the field basis $(e_\mu^\alpha, \psi_\mu, \psi_y, \chi, V_\mu^i, v_\mu, A_\mu, V_5^i, v_5, \rho, \phi)$ to the field basis $(e_\mu^\alpha, \eta_\mu, \psi_y', \chi', B_\mu^i, b_\mu, A_\mu, \varphi^i, \tau, \varphi_0, t)$, the part of the reduced lagrangian coming from eq. (3.1) reads:

$$
\begin{aligned}
e_4^{-1}\mathcal{L} = & -R_4 - \frac{1}{2}\frac{\partial_\mu t \partial^\mu t + \partial_\mu \tau \partial^\mu \tau}{t^2} - \frac{\partial_\mu \varphi_0 \partial^\mu \varphi_0 + \partial_\mu \varphi_i \partial^\mu \varphi^i}{\varphi_0^2} - \\
& - \frac{1}{4} t \left( B_{\mu\nu}^i + \varphi^i A_{\mu\nu} \right)^2 - \frac{1}{8}\frac{\varphi_0^2}{t} \left( b_{\mu\nu} + \tau A_{\mu\nu} \right)^2 - \frac{t\,\varphi_0^2}{8} A_{\mu\nu} A^{\mu\nu} - \\
& - \frac{\tau}{8} e_4^{-1} \varepsilon^{\mu\nu\rho\sigma} \left( B_{\mu\nu}^i + \varphi^i A_{\mu\nu} \right) \left( B_{\rho\sigma}^i + \varphi^i A_{\rho\sigma} \right) - \\
& - \frac{\varphi^i}{4} e_4^{-1} \varepsilon^{\mu\nu\rho\sigma} b_{\mu\nu} \left( B_{\rho\sigma}^i + \frac{1}{2} \varphi^i A_{\rho\sigma} \right) - \\
& - \frac{i}{2} \overline{\eta}_\mu^a \gamma^{\mu\nu\rho} D_\nu \eta_{\rho\, a} + \frac{i}{2} \overline{\chi}^a \gamma^\mu D_\mu \chi_a + \frac{3}{4} i\, \overline{\psi}_y^a \gamma^\mu D_\mu \psi_{y\, a} + \cdots ,
\end{aligned}
\tag{3.4}
$$

where the dots stand for interaction terms and the primes in eq. (3.3) have been suppressed. The lagrangian can be rewritten more compactly as:

$$
\begin{aligned}
e_4^{-1}\mathcal{L} = & -R_4 - \frac{1}{2}\frac{\partial_\mu t \partial^\mu t + \partial_\mu \tau \partial^\mu \tau}{t^2} - \frac{\partial_\mu \varphi_0 \partial^\mu \varphi_0 + \partial_\mu \varphi_i \partial^\mu \varphi^i}{\varphi_0^2} - \\
& - \frac{1}{4} g_{IJ}\, F_{\mu\nu}^I F^{J\,\mu\nu} - \frac{1}{8} e_4^{-1} \theta_{IJ}\, \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^I F_{\rho\sigma}^J - \\
& - \frac{i}{2} \overline{\eta}_\mu^a \gamma^{\mu\nu\rho} D_\nu \eta_{\rho\, a} + \frac{i}{2} \overline{\chi}^a \gamma^\mu D_\mu \chi_a + \frac{3}{4} i\, \overline{\psi}_y^a \gamma^\mu D_\mu \psi_{y\, a} + \cdots ,
\end{aligned}
\tag{3.5}
$$

where the field strengths $F_{\mu\nu}^I$ $(I = 1, \ldots, 7)$ are defined through

$$
F_{\mu\nu}^I = 2\, \partial_{[\mu} B_{\nu]}^I , \qquad B_\mu^I = (B_\mu^i,\, b_\mu,\, A_\mu) , \tag{3.6}
$$

and the symmetric field-dependent matrices $g_{IJ}$ and $\theta_{IJ}$ are:

$$
g_{IJ} = \begin{pmatrix} t\, \delta_{ij} & 0 & t\, \varphi_i \\ 0 & \frac{\varphi_0^2}{2t} & \frac{\tau\, \varphi_0^2}{2t} \\ t\, \varphi_j & \frac{\tau\, \varphi_0^2}{2t} & t\, \varphi_i\, \varphi^i + \frac{\tau^2\, \varphi_0^2}{2t} + \frac{t\, \varphi_0^2}{2} \end{pmatrix} , \tag{3.7}
$$

$$
\theta_{IJ} = \begin{pmatrix} \tau\, \delta_{ij} & \varphi_i & \tau\, \varphi_i \\ \varphi_j & 0 & \frac{\varphi_i\, \varphi^i}{2} \\ \tau\, \varphi_j & \frac{\varphi_i\, \varphi^i}{2} & \tau\, \varphi_i\, \varphi^i \end{pmatrix} . \tag{3.8}
$$

Notice that the metric $g_{IJ}$, which controls the vector kinetic terms, is a positive-definite and non-singular symmetric matrix for all the allowed scalar field configurations ($\varphi_0 > 0$, $t > 0$, $\varphi_i \in \mathbb{R}$ and $\tau \in \mathbb{R}$).

The reduced theory is a USp(4), $\mathcal{N} = 4$, $D = 4$ supergravity coupled to one vector multiplet. In the schematic notation of eq. (2.1), we can write its spectrum as:

$$ {}_{D=4}[1,4,6,4,2]^{\mathcal{N}=4}_{m=0} \; + \; {}_{D=4}[0,0,1,4,6]^{\mathcal{N}=4}_{m=0} \,. \tag{3.9}$$

The eight scalar fields (one coming from the metric, six coming from the vector fields and the $D = 5$ dilaton) parametrize the coset manifold:

$$ \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SO}(6,1)}{\mathrm{SO}(6)} \,. \tag{3.10}$$

The first factor is a Kähler manifold, parametrized by the complex scalar

$$ T = t + i\,\tau \,, \qquad (t,\, \tau \in \mathbb{R}) \,, \tag{3.11}$$

with a Kähler potential

$$ K = -\log(T + \overline{T}) \,, \tag{3.12}$$

which implies (remembering our convention for the Einstein term) the kinetic term:

$$ -2\, K_{\overline{T}T} (\partial_\mu \overline{T})(\partial^\mu T) = -\frac{1}{2} \frac{(\partial_\mu t)(\partial^\mu t) + (\partial_\mu \tau)(\partial^\mu \tau)}{t^2} \,. \tag{3.13}$$

The second factor is the scalar manifold of the real fields $(\varphi_0, \varphi_i)$.

The $U$-duality group of the reduced theory is $\mathrm{SU}(1,1) \times \mathrm{SO}(6,1)$. It is associated with a solvable Lie algebra, which can be decomposed as follows:

$$ \begin{aligned} \mathrm{SU}(1,1) &= \mathrm{SO}(1,1) + \mathbf{1}^+ + \mathbf{1}^- \,, \\ \mathrm{SO}(6,1) &= \mathrm{SO}(1,1) + \mathrm{SO}(5) + \mathbf{5}^+ + \mathbf{5}^- \,. \end{aligned} \tag{3.14}$$

The $U$-duality group is not completely embedded in the electric subgroup of the full duality group $\mathrm{Sp}(14,\mathbb{R})$, acting on the seven field strengths and their duals. As a result, only part of the $U$-duality group can be gauged. In particular, the transformations associated with the generators $\mathbf{1}^-$ and $\mathbf{5}^-$ do not have a well-defined action on the elementary fields. As long as the theory remains ungauged, however, $\mathrm{Sp}(14,\mathbb{R})$ transformations can be used to connect different equivalent formulations of the same theory: besides the USp(4) of this paper, known examples are the SO(4) of [30] and the SU(4) of [31].

The non-trivial global symmetries of the theory with a well-defined action on the elementary fields are the $\mathbf{5}^+ + \mathbf{1}^+$ translations, the two dilatations and the $\mathrm{SO}(5) \sim \mathrm{USp}(4)$ transformations.

The $\mathbf{5}^+ + \mathbf{1}^+$ translations derive from the $\mathrm{U}(1)^6$ gauge symmetry of the $D = 5$ theory. Their action on the 'axions' is then

$$ \tau \to \tau + \alpha_6 \,, \qquad \varphi_i \to \varphi_i + \alpha_i \,. \tag{3.15}$$

The vectors transform as

$$b_\mu \to b_\mu - \alpha_6\, A_\mu\,, \qquad B^i_\mu \to B^i_\mu - \alpha_i\, A_\mu\,, \tag{3.16}$$

where we used the definitions of $(b_\mu,\, B^i_\mu)$ and the fact that $(v_\mu,\, V^i_\mu)$ do not transform. The scalar sector is explicitly invariant. Since

$$\delta\,(b_{\mu\nu} + \tau\, A_{\mu\nu}) = 0\,, \qquad \delta\,(B^i_{\mu\nu} + \varphi^i\, A_{\mu\nu}) = 0\,, \tag{3.17}$$

it is easily shown that also the other terms are invariant, up to total derivatives coming from the Chern-Simons term.[3]

The two dilatations act on the scalar fields as follows:

$$t \to \beta t\,, \qquad \tau \to \beta\tau\,, \qquad \varphi_0 \to \gamma\varphi_0\,, \qquad \varphi_i \to \gamma\varphi_i\,, \tag{3.18}$$

and the invariance of the scalar sector is manifest. The invariance of the whole lagrangian is obtained by requiring that the vector fields transform as

$$B^i_\mu \to \beta^{-1/2}\, B^i_\mu\,, \qquad b_\mu \to \beta^{1/2}\,\gamma^{-1}\, b_\mu\,, \qquad A_\mu \to \beta^{-1/2}\,\gamma^{-1}\, A_\mu\,. \tag{3.19}$$

Notice that the double $\mathrm{SO}(1,1)$ is a symmetry of only the reduced theory, and it is not valid when the Kaluza-Klein modes are retained, unless $\gamma = \beta^{-1/2}$: in this case, the five-dimensional $\mathrm{SO}(1,1)$ symmetry of eq. (2.2) is recovered.

Finally, the invariance under the $\mathrm{SO}(5) \sim \mathrm{USp}(4)$ symmetry is manifest.

Summarizing, the reduced action is invariant under the global symmetry $\{[\mathrm{USp}(4) \times \mathrm{SO}(1,1)] \circledS \mathcal{T}^5\} \times [\mathrm{SO}(1,1) \circledS \mathcal{T}]$, with a semidirect product structure, and under the local $\mathrm{U}(1)^7$ group, with respect to which, however, no field is charged.

## 4. Generalized dimensional reduction

In the previous section we have identified the $\mathcal{N} = 4$ supergravity, coupled to one vector multiplet, obtained from the pure ungauged $D = 5$ theory by standard dimensional reduction. In particular, we have observed that the ungauged theory does not have the form of the known $\mathrm{SO}(4)$ and $\mathrm{SU}(4)$ theories, but it is equivalent to them via a duality transformation. However, as argued in [20, 4, 10, 11], inequivalent formulations of gauged supergravities can be obtained by considering different embeddings of the $U$-duality group in the full duality group $\mathrm{Sp}(14,\mathbb{R})$. We can then use the Scherk-Schwarz mechanism to obtain a new $\mathcal{N} = 4$, $D = 4$ gauged supergravity. We are going to find, in analogy with [10], two remarkable properties. First, if the twist belongs to $\mathrm{USp}(4)$ the supergravity gauging corresponds to a flat but non semisimple group, which guarantees a vanishing scalar potential at the minimum. Moreover, a four-dimensional Chern-Simons term is automatically generated by the generalized reduction, thus preserving the consistency of the theory.

[3] As will be clear later, this is the reason why the gauging of shift symmetries requires extra Chern-Simons terms to be added to the $D = 4$ lagrangian.

The Scherk-Schwarz reduction is performed by imposing generalized periodicity conditions on the fields:

$$\Phi(x, y + 2\pi r) = U \; \Phi(x, y) \,, \tag{4.1}$$

where $U$ is a constant matrix corresponding to a symmetry of the five-dimensional theory. For the moment we consider only the case $U \in \mathrm{USp}(4)$. The case of a non-compact twist $U \in \mathrm{SO}(1,1)$ will be discussed separately at the end of this section: as we will see, in such a case a positive-definite scalar potential without critical points is generated, and the theory has no $D = 4$ maximally symmetric vacuum.

Since USp(4) has rank two, we can parametrize the twist $U$ as

$$U = \exp\left[ i \left( \alpha_1 \frac{Y}{2} + \alpha_2 \frac{T_3}{2} \right) \right] , \tag{4.2}$$

where $Y$ and $T_3$ are two representative generators in the Cartan subalgebra of USp(4), whose explicit representation is given in appendix A, and $\alpha_{1,2} \in \mathbb{R}$. Following the standard procedure, we reparametrize the twisted fields in terms of periodic ones:

$$\Phi(x,y) \equiv U(y)\, \widetilde{\Phi}(x,y) \,, \qquad \widetilde{\Phi}(x,y) = \widetilde{\Phi}(x, y + 2\pi r) \,, \tag{4.3}$$

$$U(y) = \exp\left[ \frac{i\, y}{2\pi r} \left( \alpha_1 \frac{Y}{2} + \alpha_2 \frac{T_3}{2} \right) \right] = \exp\left[ iy \left( m_1 \frac{Y + T_3}{2} + m_2 \frac{Y - T_3}{2} \right) \right] \equiv e^{iyM} . \tag{4.4}$$

For simplicity, tildes will be dropped from now on, and fields will always be understood to be periodic.

With respect to the standard reduction, the generalized one produces extra terms in the $D = 4$ effective lagrangian, proportional to

$$U^{-1}(y)\, \partial_y\, U(y) = i \; M \,. \tag{4.5}$$

In our conventions, the explicit representations of the mass matrix $M$, acting respectively on the **4** and the **5** of USp(4), are:

$$M_4 = \mathrm{diag}\,(m_1\, \sigma_3 \,, m_2\, \sigma_3) \,, \tag{4.6}$$

$$M_5 = \mathrm{diag}\,[(m_1 + m_2)\, \sigma_2 \,, (m_1 - m_2)\, \sigma_2 \,, 0] \,. \tag{4.7}$$

Some of the extra contributions to the $D = 4$ lagrangian correspond to fermion mass terms. The remaining ones can be consistently organized to describe the 'gauging' of the $\mathcal{N} = 4$ theory: the upgrade of a subgroup of the global $U$-duality group of the $D = 4$ theory to a local invariance. The gauge group, however, is not the direct product of simple and abelian factors. It can be identified with the semidirect product of the $\mathrm{U}(1) \subset \mathrm{USp}(4)$ associated with the twist $U$, and four translations: $\mathrm{U}(1) \circledS \mathcal{T}^4$. The associated Lie algebra is defined by the commutation relations:

$$\left[X_{\widehat{\imath}}, X_7\right] = f^{\widehat{\jmath}}_{\ \widehat{\imath}7} X_{\widehat{\jmath}} \,, \qquad \left[X_{\widehat{\imath}}, X_{\widehat{\jmath}}\right] = 0 \,,$$
$$f^{\widehat{\jmath}}_{\ \widehat{\imath}7} = i M^{\widehat{\jmath}}_{\ \ \widehat{\imath}} \,, \tag{4.8}$$

where $\widehat{\imath} = 1 \dots 6$, $M^{\widehat{k}}{}_{\widehat{l}} = \mathrm{diag}\,[(M_5)^k{}_l, 0]$, $X_7$ is the twist generator (with vector potential $A_\mu$), and the $X_i$ are the generators of $\mathbf{5}^+$ (with vector potentials $B^i_\mu$). Notice that only four independent $X_i$ enter non-trivially in the Lie algebra, as the fifth one commutes with the twist. Moreover, the generator $X_6$ of $\mathbf{1}^+$ always remains ungauged, since it belongs to the SU$(1,1)$ sector, which does not carry charges with respect to USp$(4)$. In the gauged theory, according to the algebra of eq. (4.8), covariant derivatives replace the ordinary derivatives of the ungauged theory, and covariant field strengths are modified accordingly:

$$\begin{aligned}
\widehat{D}_\mu \varphi^i &= \partial_\mu \varphi^i - i(M_5)^i{}_j \, (B^j_\mu + \varphi^j \; A_\mu)\,, \\
\widehat{B}^i_{\mu\nu} &= (\partial_\mu B^i_\nu - \partial_\nu B^i_\mu) - i(M_5)^i{}_j \, (A_\mu B^j_\nu - B^j_\mu \; A_\nu)\,, \\
\widehat{D}_\mu \chi_a &= \partial_\mu \chi_a - i A_\mu \, (M_4)_a{}^b \, \chi_b\,, \\
\widehat{D}_\mu \eta_{\nu\ a} &= \partial_\mu \eta_{\nu\ a} - i \; A_\mu \, (M_4)_a{}^b \, \eta_{\nu\ b}\,, \\
\widehat{D}_\mu \psi_{y\ a} &= \partial_\mu \psi_{y\ a} - i \; A_\mu \, (M_4)_a{}^b \, \psi_{y\ b}\,.
\end{aligned} \tag{4.9}$$

Consequently, the transformation laws under the gauged symmetry, with local parameters $\Xi^I$ $(I = 1, \dots, 7)$, become:

$$\begin{aligned}
\delta\varphi^i &= i(M_5)^i{}_j \, (\Xi^j + \varphi^j \; \Xi^7)\,, \\
\delta B^i_\mu &= \partial_\mu \Xi^i + i(M_5)^i{}_j \, (\Xi^7 \; B^j_\mu - \Xi^j \; A_\mu)\,.
\end{aligned} \tag{4.10}$$

Under the gauged symmetry, the $\theta_{IJ}$ matrix of eq. (3.8) transforms non linearly. This would require the addition of an extra Chern-Simons term in order to guarantee the gauge invariance of the $D = 4$ theory [32], namely:

$$-\frac{2}{3} i \; d_{\widehat{\imath}\widehat{\jmath}\widehat{k}} \, M^{\widehat{k}}{}_{\widehat{l}} \, \varepsilon^{\mu\nu\rho\sigma} B^{\widehat{\imath}}_\mu B^{\widehat{l}}_\nu B^{\widehat{\jmath}}_{\rho\sigma}\,, \tag{4.11}$$

where $d_{\widehat{\imath}\widehat{\jmath}\widehat{k}}$ is a symmetric SO(5)-invariant tensor, normalized to:

$$d_{ij6} = d_{i6j} = d_{6ij} = -\frac{1}{4}\delta_{ij}\,. \tag{4.12}$$

As argued in [10], and verified explicitly in the present case, this term is automatically produced by the generalized dimensional reduction.

The detailed expression of the gauged lagrangian can be found in appendix C. We display here only its bosonic part:

$$\begin{aligned}
e_4^{-1} \mathcal{L}^{SS}_{\text{bos}} &= -R_4 - \frac{1}{2} \frac{\partial_\mu t \partial^\mu t + \partial_\mu \tau \partial^\mu \tau}{t^2} - \frac{\partial_\mu \varphi_0 \partial^\mu \varphi_0 + \widehat{D}_\mu \varphi_i \widehat{D}^\mu \varphi^i}{\varphi_0^2} - \\
&\quad - \frac{1}{4} g_{IJ} \, \widehat{F}^I_{\mu\nu} \widehat{F}^{J\ \mu\nu} - \frac{1}{8} e_4^{-1} \theta_{IJ} \, \varepsilon^{\mu\nu\rho\sigma} \widehat{F}^I_{\mu\nu} \widehat{F}^J_{\rho\sigma} - \\
&\quad - \frac{2}{3} i \; d_{\widehat{\imath}\widehat{\jmath}\widehat{k}} \, M^{\widehat{k}}{}_{\widehat{l}} \varepsilon^{\mu\nu\rho\sigma} B^{\widehat{\imath}}_\mu B^{\widehat{l}}_\nu B^{\widehat{\jmath}}_{\rho\sigma}\,.
\end{aligned} \tag{4.13}$$

For generic values of $m_1$ and $m_2$, the gauging of the four translations allows us to shift away four spin-0 fields: they are absorbed by the corresponding vectors, which acquire a mass matrix proportional to $M_5$. For $|m_1| = |m_2|$, only two spin-0 fields are absorbed by the vectors gauging the euclidean group on the plane, $\mathrm{U}(1) \circledS \mathcal{T}^2$.

In the fermionic sector, besides the covariantization of derivatives and field strengths with respect to the gauged group, mass terms appear, proportional to $M_4$, and a super-Higgs effect takes place, with the $\psi_{y_a}$ playing the role of goldstinos. This can be deduced, for instance, by looking at the inhomogeneous terms in the supersymmetry transformations of eq. (C.4).

Depending on the specific choice of the mass parameters $m_1$ and $m_2$ appearing in the twist matrix, it is possible to break supersymmetry partially ($\mathcal{N} = 2$) or completely ($\mathcal{N} = 0$). We summarize the spectrum for the different cases in our short-hand notation:

$$
\begin{aligned}
m_1 \neq m_2 = 0: &\quad [1,2,1,0,0]^{\mathcal{N}=2}_{m=0} + \left\{2 \times [0,1,2,1,0]^{\mathcal{N}=2}_{m\neq 0}\right\} + 2 \times [0,0,1,2,2]^{\mathcal{N}=2}_{m=0}\,, \\
|m_1| \neq |m_2|, m_1 m_2 \neq 0: &\quad [1_{m=0}, 4_{m\neq 0}, 4_{m\neq 0} + 3_{m=0}, 4_{m\neq 0}, 4_{m=0}]^{\mathcal{N}=0}\,, \\
|m_1| = |m_2| \neq 0: &\quad [1_{m=0}, 4_{m\neq 0}, 2_{m\neq 0} + 5_{m=0}, 4_{m\neq 0}, 6_{m=0}]^{\mathcal{N}=0}\,.
\end{aligned}
$$

In the partially broken case, beside the gravitational multiplet and two vector multiplets of $\mathcal{N} = 2$, we can recognize a massive $\mathcal{N} = 2$ BPS short multiplet of spin 3/2 [33].

The spectrum of the theory can be easily extracted from the lagrangian in eq. (C.1). For the bosonic sector, it is useful to observe that the kinetic terms for the vector fields are diagonal in the basis of the $V^{\hat{i}}_\mu = B^{\hat{i}}_\mu + \varphi^{\hat{i}} A_\mu$:

$$
-\frac{1}{4}\, t\, \overline{V}^{i}_{\mu\nu} \overline{V}^{\mu\nu}_{i} - \frac{1}{4} \frac{\varphi_0^2}{2t}\, \overline{v}_{\mu\nu} \overline{v}^{\mu\nu} - \frac{1}{4} \frac{\varphi_0^2\, t}{2}\, A_{\mu\nu} A^{\mu\nu}\,, \tag{4.14}
$$

through covariantization with respect to the vector metric $g_{IJ}$:

$$
\begin{aligned}
&\overline{V}^{\hat{i}}_{\mu\nu} \equiv 2\mathcal{D}_{[\mu} V^{\hat{i}}_{\nu]}\,, \\
&\mathcal{D}_\mu V^{\hat{i}}_\nu \equiv \partial_\mu V^{\hat{i}}_\nu - \left(\partial_\mu \varphi^{\hat{i}}\right) A_\nu + i (M_5)^{\hat{i}}{}_{\hat{j}}\, V^{\hat{j}}_\mu A_\nu = \partial_\mu V^{\hat{i}}_\nu - \left(\mathcal{D}_\mu \varphi^{\hat{i}}\right) A_\nu\,.
\end{aligned} \tag{4.15}
$$

The spectrum is twice degenerate and reads:

$$
\begin{aligned}
\text{spin } 3/2\ (\eta^{1\ldots4}_\mu): &\quad \frac{2}{t\varphi_0^2} m_{1,2}^2\,, \\
\text{spin } 1\ (V^{1\ldots4}_\mu): &\quad \frac{2}{t\varphi_0^2} (m_1 \pm m_2)^2\,, \\
\text{spin } 1/2\ (\chi^{1\ldots4}): &\quad \frac{2}{t\varphi_0^2} m_{1,2}^2\,,
\end{aligned}
$$

from which we can check the $\mathcal{N} = 4$ mass sum rule $Str\ \mathcal{M}^2 = 0$.

### 4.1 Non-compact SO(1, 1) twist

If non-compact generators are used to perform the Scherk-Schwarz reduction, non-flat gaugings are generated [1], with a positive-definite four-dimensional potential without critical points. This is the case if we use the non-compact SO(1, 1) in the $U$-duality group for the twist, in analogy with [27].



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Given the SO(1,1) field transformations in eq. (2.2), we obtain the following relations between periodic (with tildes) and non-periodic (without tildes) fields:

$$V^i_M = e^{\Lambda\, y}\ \widetilde{V}^i_M\,, \qquad v_M = e^{-2\,\Lambda\, y}\ \widetilde{v}_M\,, \qquad X = e^{\Lambda\, y}\ \widetilde{X}\,. \tag{4.16}$$

After the SO(1,1) generalized dimensional reduction, and removing the tildes as usual, we get the following $D = 4$ bosonic lagrangian:

$$\begin{aligned} e_4^{-1}\mathcal{L}^{\mathrm{SO}(1,1)}_{\mathrm{bos}} &= -R_4 - \frac{1}{2}\frac{D_\mu t D^\mu t + D_\mu \tau D^\mu \tau}{t^2} - \frac{D_\mu\varphi_0 D^\mu\varphi_0 + D_\mu\varphi_i D^\mu\varphi_i}{\varphi_0^2} - \\ &\quad -\frac{1}{4}g_{IJ}\widehat{F}^I_{\mu\nu}\widehat{F}^{J\ \mu\nu} - \frac{1}{8}e_4^{-1}\theta_{IJ}\varepsilon^{\mu\nu\rho\sigma}\widehat{F}^I_{\mu\nu}\widehat{F}^J_{\rho\sigma} - \\ &\quad -\frac{1}{6}e_4^{-1}\Lambda\,\mathcal{C}_{IJK}\,\varepsilon^{\mu\nu\rho\sigma}B^I_\mu B^J_\nu \widehat{F}^K_{\rho\sigma} - \frac{6\Lambda^2}{t\varphi_0^2}\,, \end{aligned} \tag{4.17}$$

where the coefficients $\mathcal{C}_{IJK}$ are fully symmetrized. The non-vanishing independent ones are identified by

$$\mathcal{C}_{i67} = \varphi_i\,, \qquad \mathcal{C}_{ij6} = -\delta_{ij}\,, \qquad \mathcal{C}_{677} = \varphi_i^2\,. \tag{4.18}$$

The scalar covariant derivatives are

$$\begin{aligned} D_\mu\varphi_0 &= (\partial_\mu - \Lambda A_\mu)\,\varphi_0\,, \qquad D_\mu\varphi^i = \partial_\mu\varphi^i - \Lambda\left(B^i_\mu + \varphi^i A_\mu\right)\,, \\ D_\mu t &= (\partial_\mu + 2\Lambda A_\mu)\,t\,, \qquad D_\mu\tau = \partial_\mu\tau + 2\Lambda\,(b_\mu + \tau A_\mu)\,, \end{aligned} \tag{4.19}$$

and the covariant field strengths are

$$\begin{aligned} \widehat{F}^i_{\mu\nu} &= B^i_{\mu\nu} - \Lambda\left(A_\mu B^i_\nu - B^i_\mu A_\nu\right)\,, \\ \widehat{F}^6_{\mu\nu} &= b_{\mu\nu} + 2\Lambda\,(A_\mu b_\nu - b_\mu A_\nu)\,, \\ \widehat{F}^7_{\mu\nu} &= A_{\mu\nu}\,. \end{aligned} \tag{4.20}$$

From eq. (4.17), we see that the SO(1,1) twist produces a positive-definite, non-vanishing scalar potential without critical points, which does not admit maximally symmetric $D = 4$ vacua.

In the fermionic lagrangian, all the field strengths and the scalar derivatives become covariant as described above. The fermions remain neutral with respect to these gauge interactions, but acquire a field-dependent mass term controlled by the scalar potential:

$$-\frac{1}{2}\left(\frac{6\Lambda^2}{t\varphi_0^2}\right)^{1/2}\bar{\eta}^a_\mu\widehat{\gamma}\gamma^\mu\chi_a\,. \tag{4.21}$$

The theory now has a non-abelian gauge group, which is the semidirect product of the six translations $(B^i_\mu, b_\mu)$ and the dilatation $(A_\mu)$, namely $\mathrm{SO}(1,1)\ Ⓢ\ \mathcal{T}^6$. All the seven vectors acquire a field-dependent mass term proportional to the potential, absorbing the corresponding seven spin-0 fields. The surviving scalar, associated with the ungauged SO(1,1), has a runaway behaviour described by the potential of eq. (4.17).

## 5. $\mathcal{N} = 4$, $D \geq 6$ reductions

The discussion given in section 4 can be extended to higher dimensions. Starting from a $\mathcal{N} = 4$ pure ungauged supergravity in $D$ dimensions, by generalized dimensional reduction we can obtain a minimal[4] no-scale model in $D-1$ dimensions. However, this strategy works only for $D \leq 8$, where there is a non-trivial $R$-symmetry for the twist. As an example, in this section we will show that from the generalized reduction of an ungauged $\mathcal{N} = 4$, $D = 6$ pure supergravity we can obtain some new $\mathcal{N} = 4$, $D = 5$ gauged supergravities, spontaneously broken to $\mathcal{N} = 2$ or $\mathcal{N} = 0$. These theories were not considered in a previous classification [28] that concentrated on semisimple and abelian gaugings. Analogous results have recently been found [34] for the $\mathcal{N} = 8$ and $\mathcal{N} = 2$ cases, and the present discussion will complete that analysis. Since there exist two inequivalent $\mathcal{N} = 4$, $D = 6$ supergravities, the non-chiral one, or (2,2) [35], and the chiral one, or (4,0) [36], we now discuss them in turn. We then conclude the section with a brief discussion of the remaining cases, $D = 7 \to 6$ and $D = 8 \to 7$, which complete the analysis of such gaugings for minimal $\mathcal{N} = 4$ theories.

### 5.1 Reduction of $D = 6$ (2,2) supergravity

The $R$-symmetry of the $D = 6$ (2,2) non-chiral supergravity is $\mathrm{USp}(2) \times \mathrm{USp}(2)$. The scalar manifold of the generic theory with $n$ vectors is:

$$\frac{\mathrm{SO}(4,n)}{\mathrm{SO}(n) \times \mathrm{SO}(4)} \times \mathrm{SO}(1,1)\,, \tag{5.1}$$

which reduces to $\mathrm{SO}(1,1)$ in the pure $n = 0$ case. The $U$-duality group is then $\mathrm{SO}(1,1) \times \mathrm{SO}(4)$ [with $\mathrm{SO}(4) \sim \mathrm{SO}(3) \times \mathrm{SO}(3) \sim \mathrm{USp}(2) \times \mathrm{USp}(2)$]. The two-parameter twist can be constructed with two generators of SO(4), one for each USp(2) subgroup. The (2,2) gravitational multiplet is:

$${}_{D=6}[1, (2,1) + (1,2), (2,2) + (1,1), (2,1) + (1,2), 1]^{\mathcal{N}=4}_{m=0}\,, \tag{5.2}$$

where we have made explicit the representations under $\mathrm{USp}(2) \times \mathrm{USp}(2)$, and the spin-1 (1,1) entry is actually an antisymmetric 2-form (or equivalently one self-dual and one anti-self-dual tensor).

The 2-form is inert under the $R$-symmetry, and after reduction to $D = 5$ it produces one vector and one 2-form, which in $D = 5$ can be dualized to another vector. The remaining four vector fields are all charged and participate in the gauging of the $\mathrm{U}(1) \circledS \mathcal{T}^4$ group, with the graviphoton $e_M^{\widehat{6}}$ associated with the U(1) factor. The embedding of the gauged group can be easily understood by looking at the decomposition of the $\mathcal{N} = 4$, $D = 5$ $U$-duality algebra:

$$\mathrm{SO}(1,1) \times \mathrm{SO}(5,1) \to \mathrm{SO}(1,1) + \mathrm{SO}(1,1) + \mathrm{SO}(4) + \mathbf{4}^+ + \mathbf{4}^-\,. \tag{5.3}$$

[4] As we will see below, in six dimensions there exist two different $\mathcal{N} = 4$ theories, and only one of them gives the 'minimal' $D = 5$ no-scale model.

We thus get the following $D = 5$ theories:

$$
\begin{aligned}
m_1 \neq m_2 = 0: &\quad [1,2,1,0,0]^{\mathcal{N}=2}_{m=0} + \{2 \times [0,1,2,1,0]^{\mathcal{N}=2}_{m\neq 0}\} + 2 \times [0,0,1,2,1]^{\mathcal{N}=2}_{m=0}\,, \\
|m_1| \neq |m_2|, m_1 m_2 \neq 0: &\quad [1_{m=0}, 4_{m\neq 0}, 4_{m\neq 0} + 3_{m=0}, 4_{m\neq 0}, 2_{m=0}]^{\mathcal{N}=0}\,, \\
|m_1| = |m_2| \neq 0: &\quad [1_{m=0}, 4_{m\neq 0}, 2_{m\neq 0} + 5_{m=0}, 4_{m\neq 0}, 4_{m=0}]^{\mathcal{N}=0}\,,
\end{aligned}
\tag{5.4}
$$

where the $\mathcal{N} = 2$ and $\mathcal{N} = 0$ cases are obtained by appropriate choices of the twist parameters in the two USp(2). The masses of fermions and vectors have the same dependence on the twist parameters as in the $D = 4$ case.

### 5.2 Reduction of $D = 6$ (4,0) supergravity

Pure $\mathcal{N} = 4$, $D = 6$ chiral supergravity is anomalous, and extra multiplets have to be added to obtain a consistent theory. The choice is unique [36] and consists in adding 21 antisymmetric tensor multiplets. This produces a theory with the following field content:

$$
{}_{D=6}[1,4,5^-,0,0]^{\mathcal{N}=4}_{m=0} \;+\; 21 \times \;{}_{D=6}[0,0,1^+,4,5]^{\mathcal{N}=4}_{m=0}\,. \tag{5.5}
$$

The spin-1 entries $5^-$ and $1^+$ are (anti) self-dual tensors, transforming in the **5** and in the **1** of USp(4), respectively. The theory then has 105 scalars, parametrizing the manifold:

$$
\frac{\mathrm{SO}(5,21)}{\mathrm{USp}(4) \times \mathrm{SO}(21)}\,. \tag{5.6}
$$

We could perform a Scherk-Schwarz twist with a generator in the maximal compact subgroup of the $U$-duality group, namely USp(4) $\times$ SO(21). However, we consider only a twist in the USp(4) factor, since a twist in SO(21) does not break supersymmetry. Notice also that no vectors are present in this $D = 6$ theory, and that the only spin-1 fields charged under USp(4) are anti-self-dual tensors. This means that in the present example there are no shift symmetries to be gauged. Indeed, the only gauged symmetry can be a U(1) subgroup of USp(4), via the vector potential $e_M^{\widehat{6}}$. The generalized reduction can then be performed as in the $D = 5$ case, by choosing the twist of eq. (4.4). In the $D = 5$ reduced theory, the fermions in the **4** of USp(4) acquire mass as in the $D = 4$ case. Four of the $5^-$ tensors transform into two complex $D = 5$ tensors ($2^{\mathbf{c}}$) with masses $m_1 \pm m_2$. The other 22 tensors remain massless and neutral, so that they can be dualized to $D = 5$ vectors. Finally, the scalars acquire a potential, which vanishes at its minimum. The potential has 21 flat directions, corresponding to 21 massless scalars. The other 42+42 scalars acquire instead a mass $m_1 \pm m_2$.

In terms of $D = 5$ multiplets, the spectrum can be summarized as follows:

$$
\begin{aligned}
m_1 \neq m_2 = 0: &\quad [1,2,1,0,0]^{\mathcal{N}=2}_{m=0} + [0,2,2^{\mathbf{c}},0,0]^{\mathcal{N}=2}_{m\neq 0} + 22 \times [0,0,1,2,1]^{\mathcal{N}=2}_{m=0} + \\
&\quad +21 \times [0,0,0,2,4]^{\mathcal{N}=2}_{m\neq 0}\,, \\
|m_1| \neq |m_2|, m_1 m_2 \neq 0: &\quad [1_{m=0}, 4_{m\neq 0}, 2^{\mathbf{c}}_{m\neq 0} + 23_{m=0}, 84_{m\neq 0}, 84_{m\neq 0} + 21_{m=0}]^{\mathcal{N}=0}\,, \\
|m_1| = |m_2| \neq 0: &\quad [1_{m=0}, 4_{m\neq 0}, 1^{\mathbf{c}}_{m\neq 0} + 25_{m=0}, 84_{m\neq 0}, 42_{m\neq 0} + 63_{m=0}]^{\mathcal{N}=0}\,,
\end{aligned}
\tag{5.7}
$$

where the massive fields are also charged with respect to the local U(1) $\subset$ USp(4).

We notice finally that, starting from the (anomalous) pure $D=6$ (4,0) supergravity, or performing a consistent truncation of the reduced theory, it is possible to recover the flat pure $D=5$ gauged theory of [25]. This happens because the reduction of the pure (4,0) theory does not produce extra matter in $D=5$. In particular, we can obtain the flat case of [25] by setting $m_1=m_2$. However, we can also choose $m_1\neq m_2$, which leads to two massive complex anti-self-dual tensors. In particular, the case $m_2=0$ gives partial breaking of supersymmetry. All this generalizes the minimal $\mathcal{N}=4$, $D=5$ no-scale model given in [25].

### 5.3 Reductions from $D=7$ and $D=8$

We complete the higher-dimensional case with a brief discussion of $D=7$ and $D=8$. In pure $\mathcal{N}=4$, $D=7$ supergravity, we have the global $\mathrm{USp}(2)\times\mathrm{SO}(1,1)$ group, and the gravitational multiplet is:

$$ {}_{D=7}[1,2,1_2+3,2,1]^{\mathcal{N}=4}_{m=0}\,, \tag{5.8}$$

where we made explicit the USp(2) irreducible representations. The spinors are symplectic Majorana in $D=7$, and the spin-1 singlet $(1_2)$ is a 2-form. Standard reduction gives a non-chiral USp(2), $\mathcal{N}=4$, $D=6$ supergravity[5] coupled to one vector multiplet:

$$ {}_{D=6}[1,2+2,1_2+1+3,2+2,1]^{\mathcal{N}=4}_{m=0} \;+\; {}_{D=6}[0,0,1,2+2,1+3]^{\mathcal{N}=4}_{m=0}\,, \tag{5.9}$$

with scalar manifold

$$\frac{\mathrm{SO}(4,1)}{\mathrm{SO}(4)}\times\mathrm{SO}(1,1)\,. \tag{5.10}$$

The algebra of the duality group decomposes into

$$\mathrm{SO}(1,1)+\mathrm{SO}(1,1)+\mathrm{USp}(2)+\mathbf{3}^{+}+\mathbf{3}^{-}\,. \tag{5.11}$$

After a twist in USp(2), we end up with:

$$ {}_{D=6}[1_{m=0},4_{m\neq0},(1_2)_{m=0}+2_{m=0}+2_{m\neq0},4_{m\neq0},3_{m=0}]^{\mathcal{N}=0}\,, \tag{5.12}$$

the fermions getting a mass $m$ proportional to the twist, and two vectors getting a mass $2m$, via the gauging of the semidirect product $\mathrm{U}(1)\circledS\mathcal{T}^2$.

Finally, the $D=8$ theory is globally invariant only under $\mathrm{SO}(2)\times\mathrm{SO}(1,1)$. With respect to SO(2), the gravitational multiplet can be decomposed as

$$ {}_{D=8}[1,1^{+},1_2+2,1^{-},1]^{\mathcal{N}=4}_{m=0}\,, \tag{5.13}$$

where $(1^{\pm})$ means a complex Weyl spinor charged under $\mathrm{U}(1)\sim\mathrm{SO}(2)$. As in the previous case, the reduced theory is coupled to one vector multiplet:

$$ {}_{D=7}[1,2,1_2+1+2,2,1]^{\mathcal{N}=4}_{m=0} \;+\; {}_{D=7}[0,0,1,2,1+2]^{\mathcal{N}=4}_{m=0}\,. \tag{5.14}$$

The scalar manifold is

$$\frac{\mathrm{SO}(3,1)}{\mathrm{USp}(2)}\times\mathrm{SO}(1,1)\,, \tag{5.15}$$

[5]This USp(2) is the diagonal subgroup of the full $D=6$ $R$-symmetry group, $\mathrm{USp}(2)\times\mathrm{USp}(2)$.

and the algebra of the duality group decomposes into

$$\mathrm{SO}(1,1) + \mathrm{SO}(1,1) + \mathrm{U}(1) + \mathbf{2}^+ + \mathbf{2}^- \,. \tag{5.16}$$

Again, a U(1) twist leads to the gauging of the subgroup $\mathrm{U}(1) \circledS \mathcal{T}^2$, with matter content

$$_{D=7}[1_{m=0}, 2_{m\neq 0}, (1_2)_{m=0} + 2_{m=0} + 2_{m\neq 0}, 4_{m\neq 0}, 2_{m=0}]^{\mathcal{N}=0} \,, \tag{5.17}$$

fermions of mass $m$ and vectors of mass $2m$.

## 6. $\mathcal{Z}_2$ orbifold

We have seen that generalized dimensional reduction can give either partial or complete breaking of supersymmetry, but the number of residual supersymmetries is always even, as the mechanism cannot generate chirality. However, with the additional help of an orbifold projection, it is also possible to obtain a reduced theory with an odd number of unbroken supersymmetries. It is then interesting to study how the results obtained in section 4 can be modified by orbifold projections. With only one internal dimension, it is not restrictive to consider the $\mathcal{Z}_2$ orbifold associated with the parity $y \to -y$, which leaves the classical $D=5$ action invariant. The corresponding action on the fields can be written as:

$$\Phi(x^\mu, -y) = \mathcal{Z}_2 \; \Phi(x^\mu, y) \,, \tag{6.1}$$

where for consistency $\mathcal{Z}_2$ must square to 1. In the absence of a twist, a consistent assignment of the $\mathcal{Z}_2$ parities to the fields is the following:

$$\begin{aligned} e_\mu^\alpha &: \; + \\ \rho\,, \phi\,, v_5 &: \; + \\ A_\mu\,, v_\mu &: \; - \\ V_\mu^i &: \; (+,+,-,-,-) \\ V_5^i &: \; (-,-,+,+,+) \\ \psi_{\mu_a} &: \; (+,-,+,-) \\ \psi_{y_a}\,, \chi_a &: \; (-,+,-,+) \,. \end{aligned} \tag{6.2}$$

In our conventions, the above assignment corresponds to the following $\mathcal{Z}_2$ representation:

$$\psi_{\mu_a}(-y) = \widehat{\gamma}\, Y_a{}^b \psi_{\mu_b}(y) \,, \tag{6.3}$$

where $Y$ is the $\mathrm{U}(1) \subset \mathrm{USp}(4)$ generator given in appendix A. All other parity assignments follow from eq. (6.3), and the other admissible choices for $\mathcal{Z}_2$ are physically equivalent.

The standard reduction on the orbifold $S^1/\mathcal{Z}_2$ produces an unbroken $\mathcal{N}=2$, $D=4$ theory, with one gravitational multiplet $[e_\mu^\alpha, \psi_\mu^{1,3}, V_\mu^1]$, one vector multiplet $[V_\mu^2, \psi_y^{2,4}, V_5^{3,4}]$ and one hypermultiplet $[\chi^{2,4}, (\rho, \phi, v_5, V_5^5)]$:

$$[1,2,1,0,0]_{m=0}^{\mathcal{N}=2} \; + \; [0,0,1,2,2]_{m=0}^{\mathcal{N}=2} \; + \; [0,0,0,2,4]_{m=0}^{\mathcal{N}=2} \,. \tag{6.4}$$

We are now ready to discuss what happens if the $\mathcal{Z}_2$ orbifold projection and generalized dimensional reduction are combined. For a consistent space-time interpretation, the orbifold symmetry $\mathcal{Z}_2$ and the twist $U$ must obey the relation:

$$\mathcal{Z}_2\, U\, \mathcal{Z}_2\, U = 1\,. \tag{6.5}$$

If we express the twist as $U = \exp(i\,T)$, we can identify two ways of satisfying eq. (6.5): either $[T, \mathcal{Z}_2] = 0$ or $\{T, \mathcal{Z}_2\} = 0$. In the first case, eq. (6.5) reduces to $U^2 = 1$, which discretizes the possible values of the twist $T$. In the second case, instead, eq. (6.5) is satisfied for every value of the twist.

In both cases the gauging identified in section 4 is nullified by the orbifold, since the projection removes the zero mode of $A_\mu$, which gauges USp(4) and is crucial for the non-abelian character of the algebra in eq. (4.8). However, there is still room for a smaller abelian group to be gauged. We therefore analyse the two cases in more detail.

To satisfy $[T, \mathcal{Z}_2] = 0$, we must search for generators of USp(4) that commute with $Y$, but the most general twist with these properties is the one already analysed in section 4. To satisfy also $U^2 = 1$, we must choose $\alpha_1 \pm \alpha_2 = 0$ (mod. $2\pi$), or, equivalently, $m_{1,2} = (0, \pm 1)/(2r)$. If we do so, also the twist acts like a parity, in particular $U = \mathcal{Z}_2 \cdot \mathcal{Z}_2'$, where $\mathcal{Z}_2$ is the reflection with respect to $y = 0$ and $\mathcal{Z}_2'$ the one with respect to $y = \pi$. This is equivalent to imposing a $\mathcal{Z}_2 \times \mathcal{Z}_2'$ projection on a circle of radius $2r$. Choosing for instance $m_1 = 0$, $m_2 = 1/(2r)$, the $(\mathcal{Z}_2\,, \mathcal{Z}_2')$ parity assignments will be:

$$\begin{array}{cccccc}
e_\mu^\alpha\,, \rho\,, \phi\,, v_5 & \psi_{\mu\,a} & \psi_{y\,a}\,, \chi_A & A_\mu\,, v_\mu & V_\mu^i & V_5^i \\
(+,+) & \begin{pmatrix} +,+ \\ -,- \\ +,- \\ -,+ \end{pmatrix} & \begin{pmatrix} -,- \\ +,+ \\ -,+ \\ +,- \end{pmatrix} & (-,-) & \begin{pmatrix} +,- \\ +,- \\ -,+ \\ -,+ \\ -,- \end{pmatrix} & \begin{pmatrix} -,+ \\ -,+ \\ +,- \\ +,- \\ +,+ \end{pmatrix}
\end{array}. \tag{6.6}$$

This means that the reduced theory is an $\mathcal{N} = 1$, $D = 4$ unbroken supergravity with one gravitational multiplet $[e_\mu^\alpha, \psi_{\mu\,1}]$ and two chiral multiplets $[(\psi_{y\,2}, \chi_2), (\rho, \phi, v_5, V_5^5)]$:

$$[1,1,0,0,0]_{m=0}^{\mathcal{N}=1} \;+\; 2\; \times [0,0,0,1,2]_{m=0}^{\mathcal{N}=1}\,. \tag{6.7}$$

We can finally move to the more interesting case $\{T, \mathcal{Z}_2\} = 0$. Six generators of USp(4) have this property, namely those in the coset USp(4)/[SU(2) $\times$ U(1)]. We can easily find two of these generators that commute with each other (and obviously anticommute with $Y$). Without loss of generality we choose:

$$\begin{aligned}
T_{31} &= -i\,\Gamma_{31} = 1 \otimes \sigma_2\,, \\
T_{24} &= -i\,\Gamma_{24} = \sigma_3 \otimes \sigma_2\,.
\end{aligned} \tag{6.8}$$

Our twist then reads:

$$U = \exp\left[i\left(\alpha_1 \frac{T_{31}}{2} + \alpha_2 \frac{T_{24}}{2}\right)\right]. \tag{6.9}$$

The action of the twist gives the same results as those in section 4, but with different matrices $M_{4,5}$. In particular, we have now:

$$\begin{aligned} M_4 &= \mathrm{diag}\Big(m_1\,\sigma_2\,, m_2\,\sigma_2\Big)\,, \\ M_5 &= \mathrm{diag}\Big(-(m_1+m_2)(\sigma_2)_{13}\,, (m_1-m_2)(\sigma_2)_{24}\,, (0)_5\Big)\,, \end{aligned} \tag{6.10}$$

where $(\sigma_i)_{ij}$ means a $\sigma_i$ matrix taken in the subspace $({}_{ij})$.

The $\mathcal{Z}_2$ orbifold projection removes some fields from the reduced theory, in particular the field $A_\mu$, responsible for the gauging described in section 4. However, only half of the gauged translations are removed, while the others survive. Observe that the orbifold preserves only the vectors $V_\mu^{1,2}$ and the scalars $V_5^{3,4,5}$. However, now the twist connects $1 \to 3$ and $2 \to 4$, still allowing the surviving scalars to be gauged. This can be explicitly checked in the covariant derivative for the scalars, eq. (4.9):

$$\widehat{D}_\mu \varphi^{3,4} = \partial_\mu \varphi^{3,4} - i(M_5)^{3,4}{}_{1,2}\; V_\mu^{1,2}\,. \tag{6.11}$$

The mass matrices in eq. (6.10) are non-diagonal, but their squares are:

$$\begin{aligned} M_4^2 &= \mathrm{diag}\left(m_1^2\,, m_1^2\,, m_2^2\,, m_2^2\right)\,, \\ M_5^2 &= \mathrm{diag}\left((m_1+m_2)^2\,, (m_1-m_2)^2\,, (m_1+m_2)^2\,, (m_1-m_2)^2\,, 0\right)\,. \end{aligned} \tag{6.12}$$

The $\mathcal{Z}_2$ orbifold projection removes the second and the fourth entry in $M_4$, as well as the last three entries in $M_5$. The resulting $\mathcal{N}=2$ theory is spontaneously broken either to $\mathcal{N}=1$ or to $\mathcal{N}=0$, with two independent mass parameters, and inherits the $\mathcal{N}=4$ mass sum rule $Str\,\mathcal{M}^2 = 0$.

The spectrum of the $\mathcal{N}=4$, $D=5$ theory, after the orbifold and the partial ($m_1 \neq 0, m_2 = 0$) or total ($m_1 m_2 \neq 0$) Scherk-Schwarz breaking, reads:

$$\mathcal{N}=4,\ D=5$$

$$\downarrow \quad \mathcal{Z}_2$$

$$[1,2,1,0,0]^{\mathcal{N}=2}_{m=0} + [0,0,1,2,2]^{\mathcal{N}=2}_{m=0} + [0,0,0,2,4]^{\mathcal{N}=2}_{m=0}$$

$$\downarrow \quad SS\ (m_1 \neq 0, m_2 = 0)$$

$$[1,1,0,0,0]^{\mathcal{N}=1}_{m=0} + [0,1,2,1,0]^{\mathcal{N}=1}_{m\neq 0} + 2\times[0,0,0,1,2]^{\mathcal{N}=1}_{m=0}$$

$$\downarrow \quad SS\ (m_1 m_2 \neq 0)$$

$$[1_{m=0}, 2_{m\neq 0}, 2_{m\neq 0}, 2_{m\neq 0}, 4_{m=0}]^{\mathcal{N}=0}$$

Notice that in the partially broken theory the massive multiplet is a long spin-3/2 multiplet, because this is the only possibility with $\mathcal{N}=1$ (the short one, being BPS-charged, requires an even number of supersymmetries).

This mechanism, already considered in [22], allows us to break an $\mathcal{N}=2$ supergravity, via generalized reduction, with two independent parameters, starting from the $\mathcal{Z}_2$ orbifold of an $\mathcal{N}=4$, $D=5$ theory. The corresponding $D=4$ effective theory is the minimal $\mathcal{N}=2$ no-scale model with partial breaking of [21].

If we start from $\mathcal{N}=8$, $D=5$ supergravity, we can get a $\mathcal{N}=4$, $D=4$ spontaneously broken supergravity with four independent mass parameters, recovering the results of [14] for a type-IIB supergravity compactified on a $T^6/\mathcal{Z}_2$ orientifold with fluxes.

## 7. Conclusions and outlook

In this paper we constructed the minimal $\mathcal{N}=4$ no-scale model in four dimensions, by generalized dimensional reduction of pure $\mathcal{N}=4$, $D=5$ supergravity. We explicitly derived lagrangian and transformation laws of the $D=4$ effective theory, a $\mathcal{N}=4$ supergravity where a flat, non-semisimple group is gauged, and half or all of the supersymmetries are spontaneously broken. We found that the Scherk-Schwarz reduction automatically generates the extra Chern-Simons term that must be added to the $D=4$ lagrangian to ensure gauge invariance. We also studied how this procedure extends to non-compact twists and to higher dimensions, and found new $\mathcal{N}=4$ gauged supergravities not included in earlier classifications. Finally, we discussed the consistent orbifold projections of the theory, recovering the minimal partially broken $\mathcal{N}=2$ no-scale model of refs. [21, 22].

An interesting extension of the present work would consist in performing a similar investigation with pure but gauged $\mathcal{N}=4$, $D=5$ supergravity [17, 25] as the starting point. This may lead to a better understanding of spontaneous supersymmetry breaking and boundary actions for warped compactifications, in the context of the Randall-Sundrum model [23] with an underlying $\mathcal{N}=4$ supersymmetry [24], explicitly broken to $\mathcal{N}=2$ by the $\mathcal{Z}_2$ orbifold projection.

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## A. Conventions

We specify here our conventions on USp(4) transformations and on space-time spinors.

### A.1 USp(4): group, algebra and representations

The generic USp(4) group element is defined as $4\times4$ complex matrix $U\equiv U_a{}^b$ such that:

$$U^\dagger\, U = 1\,, \qquad U^T\,\Omega\,U=\Omega\,, \tag{A.1}$$

where $\Omega \equiv \Omega^{ab}$ is for us a real, antisymmetric symplectic metric with upper indices:

$$\Omega^{ab} \equiv \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} = \mathbf{1}_{2\times 2} \otimes i\sigma^2 \,. \tag{A.2}$$

The matrix $U$ acts from the left on column, four-component vectors with lower indices, $v \equiv v_a$, as $v\,' = U\; v$, or $v_a' = U_a{}^{\;b}\; v_b$. The second equation in (A.1) expresses the invariance of the symplectic product $u^T\;\Omega\; v \equiv u_a\;\Omega^{ab}\; v_b$:

$$(u^T\;\Omega\; v)\,' = u^T\;(U^T\;\Omega\; U)\; v = u^T\;\Omega\; v \,. \tag{A.3}$$

We can then define symplectic vectors with upper indices by:

$$u^a \equiv \Omega^{ab}\; u_b \,, \tag{A.4}$$

and write the symplectic product as:

$$u^T\;\Omega\; v = u_a\;\Omega^{ab}\; v_b = -u^b\; v_b \equiv -u\,v \,. \tag{A.5}$$

As a short-hand notation for the symplectic product, we adopt the NW-SE convention on the contraction of symplectic indices:

$$u\,v \equiv u^b\; v_b \,. \tag{A.6}$$

Our convention on how to lower the indices is set by defining the inverse symplectic metric with lower indices:

$$u_a \equiv \Omega_{ab}\; u^b \qquad \Leftrightarrow \qquad \Omega_{ab}\;\Omega^{bc} = \Omega^{cb}\;\Omega_{ba} = \delta_a^c \,. \tag{A.7}$$

The generators $a \equiv a_a{}^{\;b}$ of the USp(4) Lie algebra, defined by $U = \exp(a)$, satisfy:

$$a = -a^\dagger \,, \qquad a^T\;\Omega + \Omega\; a = 0 \,. \tag{A.8}$$

Exploiting the isomorphism Spin(5) $\sim$ USp(4), the ten USp(4) generators $a$ can be identified with those of Spin(5), whose elements $(\Gamma_i)_a{}^{\;b}$ satisfy the euclidean Clifford algebra

$$\{\Gamma_i, \Gamma_j\}_a{}^{\;b} = 2\;\delta_{ij}\;\delta_a^b \,, \tag{A.9}$$

where $i,j = 1,\dots,5$ are the indices[6] of Spin(5) and $a,b = 1,\dots,4$ are those of USp(4). Notice that we can define antisymmetric $\Gamma$ matrices with upper USp(4) indices by:

$$\Gamma_i^{ac} \equiv \Omega^{ab}\;(\Gamma_i)_b{}^{\;c} \,, \qquad \Gamma_i^{ac} = -\Gamma_i^{ca} \,. \tag{A.10}$$

[6]We are entitled to use indifferently upper or lower indices of type $i,j = 1,\dots,5$; we thus move them around freely for notational convenience.

Then, the antisymmetric two-index representation of Spin(5) of dimension 10, with generators

$$\Gamma_{ij} \equiv \frac{\Gamma_i \Gamma_j - \Gamma_j \Gamma_i}{2}\,, \tag{A.11}$$

satisfies the algebra of eq. (A.8) automatically. This means that the generic tensor $S_{a_1 \dots a_n}$ transforms under USp(4) as

$$S'_{a_1 \dots a_n} = U_{a_1}{}^{b_1} \dots U_{a_n}{}^{b_n}\, S_{b_1 \dots b_n}\,, \qquad U_a{}^b = \exp\left(\frac{1}{2}\, \alpha^{ij}\, \Gamma_{ij}\right)_a{}^b\,, \tag{A.12}$$

where the $\alpha^{ij}$ are real coefficients.

The generic USp(4) algebra element with two indices can now be decomposed as

$$S_a{}^b = S\, \delta_a^b + S^i\, (\Gamma_i)_a{}^b + S^{ij}\, (\Gamma_{ij})_a{}^b\,, \tag{A.13}$$

where

$$S = \frac{1}{4} S_a{}^a\,, \qquad S^i = \frac{1}{4} S_a{}^b (\Gamma_i)_b{}^a\,, \qquad S^{ij} = -\frac{1}{2} S_a{}^b (\Gamma_{ij})_b{}^a\,. \tag{A.14}$$

The last two relations link the **5** and the **10** of USp(4) to those of SO(5).

An explicit representation of the $(\Gamma_i)_a{}^b$ appearing in (A.9), useful for discussing the gaugings and the Scherk-Schwarz twists in $\mathcal{N} = 4$, $D = 5$ supergravity is:

$$\Gamma_{i=1\dots 3} = -\sigma_2 \otimes \sigma_{i=1\dots 3}\,, \qquad \Gamma_{i=4} = \sigma_1 \otimes \mathbf{1}\,, \qquad \Gamma_{i=5} = \sigma_3 \otimes \mathbf{1}\,. \tag{A.15}$$

A convenient embedding of $U(1) \times SU(2) \subset USp(4)$ is:

$$T_1 \equiv -i\,\Gamma_{45} = -\sigma_2 \otimes \mathbf{1}\,, \qquad T_2 \equiv -i\,\Gamma_{53} = \sigma_1 \otimes \sigma_3\,, \qquad T_3 \equiv -i\,\Gamma_{34} = \sigma_3 \otimes \sigma_3\,, \tag{A.16}$$

$$Y \equiv -i\,\Gamma_{12} = \mathbf{1} \otimes \sigma_3\,, \qquad [T_i, T_j] = 2\,i\,\epsilon_{ijk}\,T_k\,, \qquad [Y, T_i] = 0\,. \tag{A.17}$$

The fields of $\mathcal{N} = 4$, $D = 5$ supergravity fall only in the **1**, **4** and **5** irreducible representations of USp(4): we denote them here by the generic symbols $\phi$, $\chi_a$ and $A_{ab}$, with $A_{ab}$ antisymmetric and $\Omega$-traceless. If we parametrize the generic transformation by

$$U = U_a{}^b = \exp\Big(\frac{i}{2}\, \alpha^{ij}\, T_{ij}\Big)_a{}^b = 1 + \frac{i}{2}\, \alpha^{ij}\, T_{ij\,a}{}^b + \cdots\,, \tag{A.18}$$

$$T_{ij\,a}{}^b = -i\Gamma_{ij\,a}{}^b\,, \tag{A.19}$$

the non-trivial field transformations read

$$\chi_a \to \chi'_a = \exp\left(\frac{i}{2}\, \alpha^{ij}\, T_{ij}\right)_a{}^b \chi_b\,, \tag{A.20}$$

$$A_{ab} \to A'_{ab} = \exp\left(\frac{i}{2}\, \alpha^{ij}\, T_{ij}\right)_a{}^c \exp\left(\frac{i}{2}\, \alpha^{hk}\, T_{hk}\right)_b{}^d A_{cd}\,. \tag{A.21}$$

Using the decomposition in eq. (A.13) and the Clifford algebra in eq. (A.9), the infinitesimal transformation for $A_{ab}$ in the **5** of USp(4) reads

$$\delta A^i = \alpha^{hk}\, (\delta_{ih}\delta_{jk} - \delta_{ik}\delta_{jh})\, A^j = i\, \alpha^{hk}\, (\mathcal{O}_{hk})_{ij} A^j\,, \tag{A.22}$$

and is just the transformation law for the fundamental representation of SO(5). We can thus exploit this further isomorphism to rewrite the finite transformation of eq. (A.21):

$$A_i \to {A_i}' = \exp(i\ \alpha^{hk}\ \mathcal{O}_{hk})_i{}^j A_j\,, \tag{A.23}$$

where $\mathcal{O}_{hk}$ are the 10 generators of SO(5). Hence the U(1)×SU(2) generators in eqs. (A.16) and (A.17) correspond to the SO(2) × SO(3) subgroup of this SO(5). A generic field $A^i$ transforming in the **5** of USp(4) can then be decomposed as

$$A^{\pm} \equiv \frac{A^1 \mp iA^2}{\sqrt{2}} \equiv A^{\frac{1\mp i\,2}{\sqrt{2}}} \sim \mathbf{1}_{\pm}\,, \qquad A^{I=3,4,5} \sim \mathbf{3}_0\,, \tag{A.24}$$

where $\mathbf{X}_c$ denotes a $\mathbf{X}$ of SU(2) with U(1) charge equal to $c$, and we have introduced another short-hand notation to identify the composition of a charged field.

Considering only two commuting generators of U(1) × SU(2) ⊂ USp(4), $Y$ and $T_3$, we can write the explicit transformation laws of the relevant fields as

$$U = e^{\frac{i}{2}(\alpha_1 Y + \alpha_2 T_3)}\,, \qquad \chi_a \to (U_4)_a{}^b\ \chi_b\,, \quad \widetilde{A}_i \to (U_5)_i{}^j \widetilde{A}_j\,, \tag{A.25}$$

where

$$U_4 = \mathrm{diag}\left(e^{\frac{i(\alpha_1+\alpha_2)}{2}}, e^{-\frac{i(\alpha_1+\alpha_2)}{2}}, e^{\frac{i(\alpha_1-\alpha_2)}{2}}, e^{-\frac{i(\alpha_1-\alpha_2)}{2}}\right), \tag{A.26}$$

$$U_5 = \mathrm{diag}\left(e^{i\alpha_1}, e^{-i\alpha_1}, e^{i\alpha_2}, e^{-i\alpha_2}, 1\right), \tag{A.27}$$

$$\widetilde{A}^T = \left(A^{\frac{1-i\,2}{\sqrt{2}}}, A^{\frac{1+i\,2}{\sqrt{2}}}, A^{\frac{3-i\,4}{\sqrt{2}}}, A^{\frac{3+i\,4}{\sqrt{2}}}, A^5\right) = (A_1^+, A_1^-, A_2^+, A_2^-, A_0)\,. \tag{A.28}$$

### A.2 Space-time and spinors

We choose a 'mostly plus' metric:

$$\eta_{AB} = \mathrm{diag}(-1,+1,+1,+1,+1)\,, \tag{A.29}$$

with Clifford algebra

$$\{\gamma^A\ ,\ \gamma^B\} = -2\,\eta^{AB}\,. \tag{A.30}$$

Curved space-time indices are denoted with $M = (\mu, 5)$, flat tangent-space indices with $A = (\alpha, \widehat{5})$. The fünfbein is $E_M^A$, its determinant $e_5 = \det\ E_M^A$. The total antisymmetric tensor $\varepsilon^{MNPQR}$ is defined in such a way that:

$$\varepsilon^{MNPQR} = e_5\ E_A^{\ M} E_B^{\ N} E_C^{\ P} E_D^{\ Q} E_E^{\ R}\ \varepsilon^{ABCDE}\,, \qquad \varepsilon^{\widehat{0}\widehat{1}\widehat{2}\widehat{3}\widehat{5}} = +1\,, \tag{A.31}$$

$$\varepsilon^{\alpha\beta\gamma\delta\widehat{5}} = \varepsilon^{\alpha\beta\gamma\delta}\,, \qquad \varepsilon^{\mu\nu\rho\sigma} = e_4\ e^{\mu}_{\alpha} e^{\nu}_{\beta} e^{\rho}_{\gamma} e^{\sigma}_{\delta}\ \varepsilon^{\alpha\beta\gamma\delta}\,. \tag{A.32}$$

Our explicit representation for the Dirac matrices is:

$$\gamma^\alpha = \begin{pmatrix} 0 & \sigma^\alpha \\ \overline{\sigma}^\alpha & 0 \end{pmatrix}, \qquad \gamma^{\widehat{5}} = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, \qquad \widehat{\gamma} = i\gamma^{\widehat{5}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{A.33}$$

$$\sigma^\alpha = (-I, \vec{\sigma})\,, \qquad \overline{\sigma}^\alpha = (-I, -\vec{\sigma})\,, \tag{A.34}$$

with

$$\gamma_M = E_M^A \gamma_A\,, \qquad \gamma^{AB} = \frac{1}{2}\left[\gamma^A,\, \gamma^B\right] = 2\,\Sigma^{AB}\,, \tag{A.35}$$

$$\gamma^{ABCDE} = -\epsilon^{ABCDE}\,, \qquad \gamma^{ABCD} = \epsilon^{ABCDE}\,\gamma_E\,, \qquad \gamma^{ABC} = \epsilon^{ABCDE}\,\Sigma_{DE}\,. \tag{A.36}$$

Connections, covariant derivatives and curvature tensors are defined as follows:

$$\begin{aligned}
\Gamma^M{}_{NR} &= \frac{1}{2}G^{MS}\left(\partial_N G_{RS} + \partial_R G_{NS} - \partial_S G_{NR}\right)\,, \\
\omega_M{}^{AB} &= 2E^{N[A}\partial_{[N}E_{M]}^{B]} - E^{N[A}E^{RB]}E_{MC}\partial_R E_N^C\,, \\
D_M E_N^A &= \partial_M E_N^A - \Gamma^R{}_{MN}E_R^A - \omega_M{}^A{}_B E_N^B = 0\,, \\
R_{MN}{}^{AB} &= 2\partial_{[M}\omega_{N]}{}^{AB} + 2\omega_{[N}{}^{AC}\omega_{M]}{}^B{}_C\,, \\
R &= R_{MN}{}^{AB}E_A{}^M E_B{}^N\,, \\
R^M{}_{NRS} &= R_{RS}{}^{BA}E_A{}^M E_{NB} = 2\partial_{[R}\Gamma^M{}_{S]N} + 2\Gamma^M{}_{T[R}\Gamma^T{}_{S]N}\,, \\
D_M\Psi &= \partial_M\Psi + \frac{1}{2}\omega_{MAB}\Sigma^{AB}\Psi\,, \qquad [D_M, D_N]\Psi = \frac{1}{2}R_{MNAB}\Sigma^{AB}\Psi\,,
\end{aligned} \tag{A.37}$$

where

$$A_{[M}B_{N]} \equiv \frac{1}{2}(A_M B_N - A_N B_M)\,. \tag{A.38}$$

The $D = 5$ charge conjugation matrix $C$ must obey, in any conventions, the following two general properties:

$$C\,\gamma^A\,C^{-1} = (\gamma^A)^T\,, \qquad C^T = -C\,. \tag{A.39}$$

We can then formulate the symplectic Majorana condition on spinors as

$$\psi^a = C\,\overline{\psi_a}^T = C(\overline{\psi}^T)^a\,, \tag{A.40}$$

where in writing the second equality we have exploited our previous conventions for raising and lowering symplectic indices. From the above conventions, the following hermiticity relations for symplectic Majorana spinors follow:

$$\overline{\psi}^a\,\gamma^{A_1}\ldots\gamma^{A_n}\,\chi_a = -\overline{\chi}^a\,\gamma^{A_n}\ldots\gamma^{A_1}\,\psi_a = -(\overline{\psi}^a\,\gamma^{A_1}\ldots\gamma^{A_n}\,\chi_a)^\dagger\,. \tag{A.41}$$

Notice also that the symplectic metric acts like the charge conjugation matrix in the symplectic space of the $\Gamma_i$:

$$\Omega\,\Gamma_i\,\Omega^{-1} = \Gamma_i^T\,, \qquad \Omega = -\Omega^T\,, \tag{A.42}$$

so that:

$$\overline{\psi}^a(\Gamma_{i_1}\ldots\Gamma_{i_n})_a{}^b\chi_b = -\overline{\chi}^a(\Gamma_{i_n}\ldots\Gamma_{i_1})_a{}^b\psi_b = -[\overline{\psi}^a(\Gamma_{i_1}\ldots\Gamma_{i_n})_a{}^b\chi_b]^\dagger\,. \tag{A.43}$$

Then, all fermionic bilinears of the form $\overline{\psi}^a(\Gamma_{i_1}\ldots\Gamma_{i_n})_a{}^b\,\gamma^{A_1}\ldots\gamma^{A_n}\,\chi_b$ are anti-hermitian.

## B. Ungauged $\mathcal{N}=4$, $D=5$ supergravity

In our conventions, and neglecting four-fermion terms that are not relevant to the present work, the ungauged $\mathcal{N}=4$, $D=5$ lagrangian for pure supergravity [16, 17] reads:

$$
\begin{aligned}
e_5^{-1}\mathcal{L} = & -R_5 - \frac{1}{2}\partial_M\phi\,\partial^M\phi - \frac{1}{4}X^4 v_{MN}v^{MN} - \frac{1}{4}X^{-2}V^i_{MN}V_i^{MN} - \\
& -\frac{1}{8}e_5^{-1}\varepsilon^{MNRST}V^i_{MN}V^i_{RS}v_T - \frac{i}{2}\overline{\psi}^a_M\gamma^{MNR}D_N\psi_{R\,a} + \frac{i}{2}\overline{\chi}^a\gamma^M D_M\chi_a - \\
& -\frac{i}{8\sqrt{2}}\left(X^{-1}V^i_{MN}\Gamma_{i\,a}{}^b + \frac{1}{\sqrt{2}}X^2\delta_a{}^b v_{MN}\right)\overline{\psi}^a_R\gamma^{[R}\gamma^{MN}\gamma^{S]}\psi_{S\,b} - \\
& -\frac{i}{4\sqrt{6}}\left(X^{-1}V^i_{MN}\Gamma_{i\,a}{}^b - \sqrt{2}X^2\delta_a{}^b v_{MN}\right)\overline{\psi}^a_R\gamma^{MN}\gamma^R\chi_b + \\
& +\frac{i}{24\sqrt{2}}\left(X^{-1}V^i_{MN}\Gamma_{i\,a}{}^b - \frac{5}{\sqrt{2}}X^2\delta_a{}^b v_{MN}\right)\overline{\chi}^a\gamma^{MN}\chi_b + \\
& +\frac{i}{2\sqrt{2}}\partial_N\phi\,\overline{\psi}^a_M\gamma^N\gamma^M\chi_a\,,
\end{aligned}
\tag{B.1}
$$

where:

$$
X = \exp\left(-\frac{\phi}{\sqrt{6}}\right), \qquad V^i_{MN} = 2\,\partial_{[M}V^i_{N]}, \qquad v_{MN} = 2\,\partial_{[M}v_{N]}\,. \tag{B.2}
$$

Neglecting three-fermion terms, the supersymmetry transformation laws are:

$$
\begin{aligned}
\delta E_M^A &= \frac{i}{4}\overline{\psi}^a_M\gamma^A\epsilon_a\,,\\
\delta V^i_M &= \frac{i}{2\sqrt{2}}X\left(\overline{\psi}^a_M + \frac{1}{\sqrt{3}}\overline{\chi}^a\gamma_M\right)\Gamma^i{}_a{}^b\epsilon_b\,,\\
\delta v_M &= \frac{i}{4}X^{-2}\left(\overline{\psi}^a_M - \frac{2}{\sqrt{3}}\overline{\chi}^a\gamma_M\right)\epsilon_a\,,\\
\delta\phi &= \frac{i}{2\sqrt{2}}\overline{\chi}^a\epsilon_a\,,\\
\delta\psi_{M\,a} &= D_M\epsilon_a - \frac{1}{12\sqrt{2}}\left(\gamma_M{}^{NR} + 4\delta_M^N\gamma^R\right)\left(X^{-1}V^i_{NR}\Gamma_{i\,a}{}^b + \frac{1}{\sqrt{2}}X^2\delta_a{}^b v_{NR}\right)\epsilon_b\,,\\
\delta\chi_a &= \frac{1}{2\sqrt{2}}\gamma^M\partial_M\phi\,\epsilon_a + \frac{1}{4\sqrt{6}}\gamma^{MN}\left(X^{-1}V^i_{MN}\Gamma_{i\,a}{}^b - \sqrt{2}X^2\delta_a{}^b v_{MN}\right)\epsilon_b\,.
\end{aligned}
\tag{B.3}
$$

## C. The $\mathcal{N}=4$, $D=4$ reduced supergravity

We give here lagrangian and transformation laws for the fully broken $D=4$ effective theory, obtained via generalized reduction from the $\mathcal{N}=4$, $D=5$ ungauged theory. The unbroken theory corresponding to the standard reduction can be easily extracted by setting the Scherk-Schwarz twist $M=0$, while the partially broken theory can be obtained by choosing $|m_1| \neq |m_2| = 0$ in $M$. In the broken cases one can move to the unitary gauge, removing the goldstinos according to the standard procedure. Symplectic indices are suppressed according to the NW–SE convention. Neglecting as before four-fermion terms, the lagrangian reads:

$$
\mathcal{L}^{SS} = \mathcal{L}^{SS}_{\text{bos}} + \mathcal{L}^{SS}_{\text{fer}}\,, \tag{C.1}
$$

with

$$
\begin{aligned}
e_4^{-1}\mathcal{L}^{SS}_{\rm bos} = & -R_4 - \frac{1}{2}\frac{\partial_\mu t\partial^\mu t + \partial_\mu\tau\partial^\mu\tau}{t^2} - \frac{\partial_\mu\varphi_0\partial^\mu\varphi_0 + \widehat{D}_\mu\varphi_i\widehat{D}^\mu\varphi^i}{\varphi_0^2} - \\
& -\frac{1}{4}\, g_{IJ}\,\widehat{F}^I_{\mu\nu}\widehat{F}^{J\ \mu\nu} - \frac{1}{8}\, e_4^{-1}\theta_{IJ}\,\varepsilon^{\mu\nu\rho\sigma}\widehat{F}^I_{\mu\nu}\widehat{F}^J_{\rho\sigma} - \frac{2}{3}i\; d_{\widehat{\imath}\widehat{\jmath}\widehat{k}}\, M^{\widehat{k}}{}_{\widehat{l}}\,\varepsilon^{\mu\nu\rho\sigma}B^{\widehat{\imath}}_\mu B^{\widehat{l}}_\nu B^{\widehat{\jmath}}_{\rho\sigma}\,, \quad \text{(C.2)}
\end{aligned}
$$

$$
\begin{aligned}
e_4^{-1}\mathcal{L}^{SS}_{\rm fer} = & -\frac{1}{2}e_4^{-1}\varepsilon^{\mu\nu\rho\sigma}\overline{\eta}_\mu\widehat{\gamma}\gamma_\nu\left(\widehat{D}_\sigma + \frac{1}{2}\frac{\sqrt{2}}{t^{1/2}\varphi_0}\widehat{\gamma}\gamma_\sigma M_4\right)\eta_\rho + \frac{1}{2}i\overline{\chi}\left(\gamma^\mu\widehat{D}_\mu + \frac{\sqrt{2}}{t^{1/2}\varphi_0}\widehat{\gamma}M_4\right)\chi + \\
& + \frac{3}{4}i\;\overline{\psi}_y\left(\gamma^\mu\widehat{D}_\mu + 2\frac{\sqrt{2}}{t^{1/2}\varphi_0}\widehat{\gamma}M_4\right)\psi_y + \frac{3}{2}\frac{\sqrt{2}}{t^{1/2}\varphi_0}\overline{\psi}_y\gamma^\mu M_4\;\eta_\mu - \\
& -\frac{1}{8}\frac{t^{\frac{1}{2}}\varphi_0}{\sqrt{2}}\overline{\eta}_\mu\left[\left(\frac{\widehat{B}^i_{\rho\sigma} + A_{\rho\sigma}\varphi^i}{\varphi_0}\Gamma_i + \frac{b_{\rho\sigma} + A_{\rho\sigma}\tau}{2t}\right)\widehat{\gamma} - \frac{i}{2}A_{\rho\sigma}\right] \times \\
& \times\left(e_4^{-1}\varepsilon^{\mu\nu\rho\sigma} + 2ig^{\mu\rho}g^{\nu\sigma}\widehat{\gamma}\right)\eta_\nu - \\
& -\frac{1}{4}\frac{t^{\frac{1}{2}}\varphi_0}{\sqrt{2}}\overline{\psi}_y\left[\left(\frac{\widehat{B}^{\mu\nu}_i + A^{\mu\nu}\varphi_i}{\varphi_0}\Gamma_i + \frac{b^{\mu\nu} + A^{\mu\nu}\tau}{2t}\right) - \frac{3i}{2}A^{\mu\nu}\widehat{\gamma}\right]\Sigma_{\mu\nu}\psi_y + \\
& + \frac{i}{12}\frac{t^{\frac{1}{2}}\varphi_0}{\sqrt{2}}\overline{\chi}\left[\left(\frac{\widehat{B}^{\mu\nu}_i + A^{\mu\nu}\varphi_i}{\varphi_0}\Gamma_i - \frac{5}{2}\frac{b^{\mu\nu} + A^{\mu\nu}\tau}{t}\right) + \frac{3i}{2}A^{\mu\nu}\widehat{\gamma}\right]\Sigma_{\mu\nu}\chi - \\
& -\frac{1}{8}\frac{t^{\frac{1}{2}}\varphi_0}{\sqrt{2}}\overline{\psi}_y\left[\left(\frac{\widehat{B}^i_{\rho\sigma} + A_{\rho\sigma}\varphi^i}{\varphi_0}\Gamma_i + \frac{b_{\rho\sigma} + A_{\rho\sigma}\tau}{2t}\right)\widehat{\gamma} + \frac{3i}{2}A_{\rho\sigma}\right]\gamma^\mu\gamma^{\rho\sigma}\eta_\mu - \\
& -\frac{i}{2\sqrt{3}}\frac{t^{\frac{1}{2}}\varphi_0}{\sqrt{2}}\overline{\eta}_\mu\left(\frac{\widehat{B}^i_{\rho\sigma} + A_{\rho\sigma}\varphi^i}{\varphi_0}\Gamma_i - \frac{5}{2}\frac{b_{\rho\sigma} + A_{\rho\sigma}\tau}{t}\right)\Sigma^{\rho\sigma}\gamma^\mu\chi - \\
& -\frac{1}{2\sqrt{3}}\frac{t^{\frac{1}{2}}\varphi_0}{\sqrt{2}}\overline{\psi}_y\left(\frac{\widehat{B}^{\mu\nu}_i + A^{\mu\nu}\varphi_i}{\varphi_0}\Gamma_i - \frac{5}{2}\frac{b^{\mu\nu} + A^{\mu\nu}\tau}{t}\right)\widehat{\gamma}\Sigma_{\mu\nu}\chi + \\
& + \frac{i}{4}\overline{\eta}_\mu\left(\frac{\widehat{D}_\sigma\varphi^i}{\varphi_0}\Gamma_i + \frac{\partial_\sigma\tau}{2t}\right)e_4^{-1}\varepsilon^{\mu\nu\rho\sigma}\gamma_\nu\eta_\rho + \frac{1}{8}\overline{\psi}_y\left(\frac{\widehat{D}_\mu\varphi^i}{\varphi_0}\Gamma_i + \frac{\partial_\mu\tau}{2t}\right)\widehat{\gamma}\gamma^\mu\psi_y - \\
& -\frac{1}{12}\overline{\chi}\left(\frac{\widehat{D}_\mu\varphi^i}{\varphi_0}\Gamma_i - \frac{5}{2}\frac{\partial_\mu\tau}{t}\right)\widehat{\gamma}\gamma^\mu\chi - \\
& -\frac{i}{2}\overline{\psi}_y\left(\frac{\widehat{D}_\nu(\varphi^i\Gamma_i - i\varphi_0\widehat{\gamma})}{\varphi_0} + \frac{\partial_\nu(\tau - it\widehat{\gamma})}{2t}\right)\gamma^\mu\gamma^\nu\eta_\mu + \\
& + \frac{i}{2\sqrt{3}}\overline{\eta}_\mu\left(\frac{\partial_\nu(t + i\tau\widehat{\gamma})}{t} - \frac{\widehat{D}_\nu(\varphi_0 + i\varphi^i\Gamma_i\widehat{\gamma})}{\varphi_0}\right)\gamma^\nu\gamma^\mu\chi + \\
& + \frac{i}{\sqrt{3}}\overline{\psi}_y\left(\frac{\widehat{D}_\mu\varphi^i}{\varphi_0}\Gamma_i - \frac{\partial_\mu\tau}{t}\right)\gamma^\mu\chi\,. \quad \text{(C.3)}
\end{aligned}
$$

The explicit expressions for the covariant derivatives and the four-dimensional fields are given in the text. The corresponding supersymmetry transformations are, up to three-fermion terms:

$$
\delta e^\alpha_\mu = \frac{i}{4}\overline{\eta}_\mu\gamma^\alpha\epsilon\,,
$$

$$
\begin{aligned}
\frac{\delta t}{t} &= \frac{1}{4}\left(\overline{\psi}_y\widehat{\gamma} + \frac{2i}{\sqrt{3}}\overline{\chi}\right)\epsilon\,, \qquad \frac{\delta\tau}{t} = \frac{i}{4}\left(\overline{\psi}_y + \frac{2i}{\sqrt{3}}\overline{\chi}\widehat{\gamma}\right)\epsilon\,,\\
\frac{\delta\varphi_0}{\varphi_0} &= \frac{1}{4}\left(\overline{\psi}_y\widehat{\gamma} - \frac{i}{\sqrt{3}}\overline{\chi}\right)\epsilon\,, \qquad \frac{\delta\varphi^i}{\varphi_0} = \frac{i}{4}\left(\overline{\psi}_y - \frac{i}{\sqrt{3}}\overline{\chi}\widehat{\gamma}\right)\Gamma^i\,\epsilon\,,\\
\frac{\varphi_0 t^{1/2}}{\sqrt{2}}\delta A_\mu &= \frac{1}{4}\overline{\eta}_\mu\widehat{\gamma}\epsilon\,,\\
\frac{\varphi_0 t^{1/2}}{\sqrt{2}}\delta b_\mu &= \frac{i}{4}\overline{\eta}_\mu\left(t + i\tau\widehat{\gamma}\right)\epsilon + \frac{t}{8}\left(\overline{\psi}_y\widehat{\gamma} - \frac{4i}{\sqrt{3}}\overline{\chi}\right)\gamma_\mu\epsilon\,,\\
\frac{\varphi_0 t^{1/2}}{\sqrt{2}}\delta B^i_\mu &= \frac{i}{4}\overline{\eta}_\mu\left(\varphi_0\Gamma^i + i\varphi^i\widehat{\gamma}\right)\epsilon + \frac{\varphi_0}{8}\left(\overline{\psi}_y\widehat{\gamma} + \frac{2i}{\sqrt{3}}\overline{\chi}\right)\gamma_\mu\Gamma^i\,\epsilon\,,\\
\delta\eta_\mu &= \widehat{D}_\mu\epsilon + \frac{1}{2}\frac{\sqrt{2}}{t^{1/2}\varphi_0}\widehat{\gamma}\gamma_\mu M_4\;\epsilon + \frac{i}{2}\widehat{\gamma}\left(\frac{\widehat{D}_\mu\varphi^i}{\varphi_0}\Gamma_i + \frac{1}{2}\frac{\partial_\mu\tau}{t}\right)\epsilon -\\
&\quad - \frac{1}{8}\frac{\varphi_0 t^{1/2}}{\sqrt{2}}\left(\gamma_\mu{}^{\nu\rho} + 2\delta^\nu_\mu\gamma^\rho\right)\left(\frac{\widehat{B}^i_{\nu\rho} + \varphi^i A_{\nu\rho}}{\varphi_0}\Gamma^i + \frac{b_{\nu\rho} + \tau A_{\nu\rho}}{2t} - \frac{i}{2}\widehat{\gamma}A_{\nu\rho}\right)\epsilon\\
\delta\psi_y &= i\frac{\sqrt{2}}{t^{1/2}\varphi_0}M_4\;\epsilon + \frac{i}{3}\left(\frac{\partial_\mu(t - i\widehat{\gamma}\tau)}{2t} + \frac{\widehat{D}_\mu(\varphi_0 - i\widehat{\gamma}\varphi^i\Gamma_i)}{\varphi_0}\right)\widehat{\gamma}\gamma^\mu\epsilon +\\
&\quad + \frac{i}{12}\frac{\varphi_0 t^{1/2}}{\sqrt{2}}\widehat{\gamma}\gamma^{\mu\nu}\left(\frac{b_{\mu\nu} + \tau A_{\mu\nu}}{2t} + \frac{\widehat{B}^i_{\mu\nu} + \varphi^i A_{\mu\nu}}{\varphi_0}\Gamma^i - \frac{3}{2}i\widehat{\gamma}A_{\mu\nu}\right)\epsilon\,,\\
\delta\chi &= \frac{1}{2\sqrt{3}}\left(\frac{\partial_\mu(t - i\widehat{\gamma}\tau)}{t} - \frac{\widehat{D}_\mu(\varphi_0 - i\widehat{\gamma}\varphi^i\Gamma_i)}{\varphi_0}\right)\gamma^\mu\epsilon +\\
&\quad + \frac{1}{4\sqrt{3}}\frac{\varphi_0 t^{1/2}}{\sqrt{2}}\left(\frac{\widehat{B}^i_{\mu\nu} + \varphi^i A_{\mu\nu}}{\varphi_0}\Gamma^i - \frac{b_{\mu\nu} + \tau A_{\mu\nu}}{t}\right)\gamma^{\mu\nu}\epsilon\,,
\end{aligned}
\tag{C.4}
$$

where the infinitesimal parameter $\epsilon$ has been rescaled as

$$\epsilon_{D=5} = \rho^{-1/4}\;\epsilon_{D=4}\,, \tag{C.5}$$

and its covariant derivative is defined as

$$\widehat{D}_\mu\epsilon_a = \partial_\mu\epsilon_a - iA_\mu\;(M_4)_a{}^b\;\epsilon_b\,. \tag{C.6}$$

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